

Polynomial Functions

POLYNOMIAL FUNCTION	$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ Degree n , leading coefficient a_n .
TRANSFORMED POWER FUNCTION	$y = a[k(x - d)]^n + c$
EVEN FUNCTION	$f(-x) = f(x)$ Symmetric about the y -axis.
ODD FUNCTION	$f(-x) = -f(x)$ Symmetric about the origin.
AVERAGE RATE OF CHANGE	$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$
INSTANTANEOUS RATE OF CHANGE (ESTIMATE)	$\frac{f(a + h) - f(a)}{h}$ With h very small.
FINITE DIFFERENCES	$\Delta^n y = a \cdot n!$ For degree n and equal x steps of 1, the n th differences are constant.

Polynomial Equations

DIVISION STATEMENT	$P(x) = D(x)Q(x) + R(x)$
REMAINDER THEOREM	$P(x) \div (x - a)$ leaves remainder $P(a)$ Dividing by $(ax - b)$ leaves $P(\frac{b}{a})$.
FACTOR THEOREM	$(x - a)$ is a factor of $P(x) \iff P(a) = 0$
RATIONAL ZERO TEST	$x = \frac{b}{a}$ b divides the constant term, a divides the leading coefficient.
SUM OF CUBES	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
DIFFERENCE OF CUBES	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Rational Functions

RECIPROCAL FUNCTION	$y = \frac{1}{f(x)}$ Vertical asymptotes where $f(x) = 0$; invariant points where $f(x) = \pm 1$.
LINEAR OVER LINEAR	$y = \frac{ax + b}{cx + d}$ Vertical asymptote $x = -\frac{d}{c}$, horizontal asymptote $y = \frac{a}{c}$.
HORIZONTAL ASYMPTOTE	$\deg N < \deg D : y = 0 \qquad \deg N = \deg D : y = \frac{a}{b}$ a and b are the leading coefficients of N and D . When $\deg N = \deg D + 1$ there is an oblique asymptote instead: divide.
HOLE	$f(x) = \frac{(x - a)g(x)}{(x - a)h(x)}$ A factor that cancels gives a hole at $x = a$, not an asymptote.

Trigonometric Ratios

RADIANS AND DEGREES	$\pi \text{ rad} = 180^\circ$
ARC LENGTH	$a = r\theta$ θ in radians.
SPECIAL ANGLES	$\sin \frac{\pi}{6} = \frac{1}{2} \qquad \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$
RECIPROCAL RATIOS	$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$
QUOTIENT IDENTITY	$\tan x = \frac{\sin x}{\cos x}$
PYTHAGOREAN IDENTITY	$\sin^2 x + \cos^2 x = 1$

PYTHAGOREAN IDENTITIES, TAN AND COT	$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$
CO-FUNCTION IDENTITIES	$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \cos x = \sin\left(\frac{\pi}{2} - x\right)$
SHIFT BY A QUARTER TURN	$\sin\left(x + \frac{\pi}{2}\right) = \cos x \qquad \cos\left(x + \frac{\pi}{2}\right) = -\sin x$
COMPOUND ANGLE, SINE	$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
COMPOUND ANGLE, COSINE	$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
DOUBLE ANGLE	$\sin 2x = 2 \sin x \cos x \qquad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
DOUBLE ANGLE, COSINE	$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$

Trigonometric Functions

SINUSOIDAL FUNCTION	$y = a \sin[k(x - d)] + c \qquad y = a \cos[k(x - d)] + c$
PERIOD	period $= \frac{2\pi}{ k }$ For $y = \tan kx$ the period is $\frac{\pi}{ k }$.
AMPLITUDE AND AXIS	$ a = \frac{\text{max} - \text{min}}{2} \qquad c = \frac{\text{max} + \text{min}}{2}$
ASYMPTOTES OF TANGENT	$y = \tan x : \quad x = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$

Exponential & Logarithmic Functions

DEFINITION OF A LOGARITHM	$y = b^x \iff x = \log_b y$ For $b > 0, b \neq 1, y > 0$.
PRODUCT LAW	$\log_b(MN) = \log_b M + \log_b N$
QUOTIENT LAW	$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$
POWER LAW	$\log_b M^n = n \log_b M$
SPECIAL VALUES	$\log_b 1 = 0 \qquad \log_b b = 1 \qquad b^{\log_b x} = x$
CHANGE OF BASE	$\log_b x = \frac{\log x}{\log b}$
HALF-LIFE	$A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T}}$
COMPOUND GROWTH	$A = P(1 + i)^n$
PH	$\text{pH} = -\log [\text{H}^+]$
EARTHQUAKE MAGNITUDE	$M = \log\left(\frac{I}{I_0}\right) \qquad M_2 - M_1 = \log\left(\frac{I_2}{I_1}\right)$
SOUND LEVEL (DECIBELS)	$\beta = 10 \log\left(\frac{I}{I_0}\right) \qquad \beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$

Combining Functions

SUM AND DIFFERENCE	$(f \pm g)(x) = f(x) \pm g(x)$ Domain: where both f and g are defined.
PRODUCT	$(f \cdot g)(x) = f(x)g(x)$
QUOTIENT	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ Also exclude every x where $g(x) = 0$.
COMPOSITION	$(f \circ g)(x) = f(g(x))$ x in the domain of g and $g(x)$ in the domain of f .
INVERSE CHECK	$(f \circ f^{-1})(x) = x$